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A FORMULATION OF SNELL'S LAW AND THE DIVERGENCE LAW FOR RAY ACO--ETC(U)
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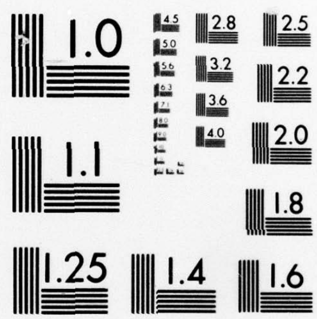

MICROCOPY RESOLUTION TEST CHART
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University Park, Pennsylvania

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A FORMULATION OF SNELL'S LAW AND THE DIVERGENCE LAW FOR RAY ACOUSTICS WITH A BIVARIATE SOUND VELOCITY FUNCTION.

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Abstract: The sound velocity, V, is a function of the horizontal and depth coordinates. The independent coordinate curves are taken as the ray paths, (θ_o = const.), and wave fronts, (t = const.); they are assumed to intersect at right angles. Two equations are derived in terms of the arc length along the ray (s) and along the wave front (σ) and the angle to the horizontal of the ray (θ):

$$\frac{\partial^2 s}{\partial\theta_o \partial t} = -\frac{\partial\sigma}{\partial\theta_o}\frac{\partial\theta}{\partial t}$$

$$\frac{\partial^2 \sigma}{\partial t \partial\theta_o} = \frac{\partial s}{\partial t}\frac{\partial\theta}{\partial\theta_o}$$

The first of these yields the Snell's Law form for bivariate V, Equation (8); while the second yields a differential equation (10) governing ray divergence ($\partial\sigma/\partial\theta_o$) as a function of t. It, with Equations (2), (7) and (9), can be used to compute a raypath and its intensity as a function of time, (t).

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Consider the following functions and geometry of the acoustic raypaths and intensity problem:

x : the abscissa coordinate with positive taken toward the right; particularly, the horizontal distance traveled by the wave front at time, t.

z : the ordinate coordinate with positive taken downward; particularly, the depth of the wave front ($z = 0$ at the water's surface) at time t.

t : the time of travel along the ray path of the wave front from the time of acoustic transmission, $t = 0$.

z_o : the initial depth of the transducer (thought of as a point) and, therefore, of the pencil of rays; it is held fixed.

V : the sound velocity function which in this paper depends on z and x, where the x dependence is so chosen as to make $x = 0$ correspond to the abscissa of the transducer.

θ : the angle measured clockwise from the horizontal to the tangent of the ray at time, t.

θ_o : θ at $t = 0$.

ψ : the angle measured clockwise from the downward vertical to the tangent of the wave front at time, t.

s : the arc length along the ray path.

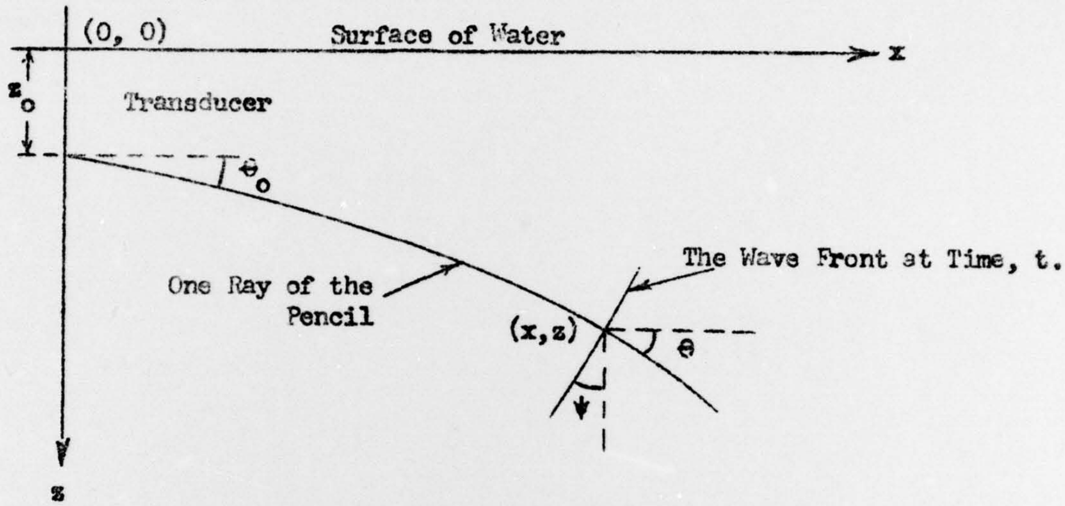
The functional dependence of the variables defined is as follows with respect to the complete pencil of rays through the transducer:

Independent Variables: t, θ_o, z_o (held constant)

Dependent Variables: x, z, θ, ψ; on all independent variables

: $V = V(z, x)$; on the coordinates

The accompanying picture is:



The following equations evidently hold with respect to the coordinates of the raypath at time, t:

$$\frac{\partial s}{\partial t} = V \tag{1}$$

$$\frac{\partial x}{\partial t} = V \cos \theta \quad ; \quad \frac{\partial z}{\partial t} = V \sin \theta \tag{2}$$

Next, let us define analogous equationsto (1) and (2) with respect to the wave-front curve. Along the wavefront t is held constant and θ_o is allowed to vary just as along the ray θ_o is held constant and t allowed to vary. Therefore, defining σ:

$\sigma \equiv$ arc length along the wavefront curve;

the analog to equation (1) is

$$\frac{\partial \sigma}{\partial \theta_o} = Q(t, \theta_o) \tag{3}$$

where Q is defined by Equation (3). From this and the geometric picture, the analogs to (2) are determined as

$$\frac{\partial x}{\partial \theta_o} = -Q \sin \psi \quad ; \quad \frac{\partial z}{\partial \theta_o} = Q \cos \psi \tag{4}$$

Sufficient differentiability conditions are imposed such that the following hold:

$$\frac{\partial^2 x}{\partial t \partial \theta_o} = \frac{\partial^2 x}{\partial \theta_o \partial t} \quad ; \quad \frac{\partial^2 z}{\partial t \partial \theta_o} = \frac{\partial^2 z}{\partial \theta_o \partial t}$$

These yield the following equivalent conditions:

$$\frac{-\partial Q}{\partial t} \sin \psi - Q \cos \psi \frac{\partial \psi}{\partial t} = \frac{\partial V}{\partial \theta_o} \cos \theta - V \sin \theta \frac{\partial \theta}{\partial \theta_o}$$

$$\frac{\partial Q}{\partial t} \cos \psi - Q \sin \psi \frac{\partial \psi}{\partial t} = \frac{\partial V}{\partial \theta_o} \sin \theta + V \cos \theta \frac{\partial \theta}{\partial \theta_o}$$

If rewritten in the form

$$\left(\frac{-\partial Q}{\partial t} \sin \psi + V \frac{\partial \theta}{\partial \theta_o} \sin \theta\right) - \left(\frac{\partial V}{\partial \theta_o} \cos \theta + Q \frac{\partial \psi}{\partial t} \cos \psi\right) = 0$$

$$\left(\frac{\partial Q}{\partial t} \cos \psi - V \frac{\partial \theta}{\partial \theta_o} \cos \theta\right) - \left(\frac{\partial V}{\partial \theta_o} \sin \theta + Q \frac{\partial \psi}{\partial t} \sin \psi\right) = 0$$

it can be seen that Equations (5) and (6),

$$\frac{\partial Q}{\partial t} = V \frac{\partial \theta}{\partial \theta_o} \tag{5}$$

$$\frac{\partial V}{\partial \theta_o} = -Q \frac{\partial \psi}{\partial t} \tag{6}$$

are necessary and sufficient conditions that $\theta = \psi$. The physical interpretation of $\theta = \psi$ is that the wave front is always at right angles to the ray path. While one might endeavor to prove from another principle that such indeed is the physical relationship between raypaths and wave fronts, the hypothesis that sound ray paths are generated normal to the wave front is sufficiently succinct to merit acceptance in its own right. Indeed, if one were formulating the problem in reverse, that is, given a set of wave fronts, construct the path of a particle being swept along by the wave, the paths constructed would be computed by making them normal to the wave fronts.

With the assumption, $\theta = \psi$, Equations (4) become (4')

$$\frac{\partial x}{\partial \theta_o} = -Q \sin \theta \; ; \; \frac{\partial z}{\partial \theta_o} = Q \cos \theta \qquad (4')$$

and it can be shown that Equation (6') is the Snell's Law Equation for $V(z, x)$:

$$\frac{\partial V}{\partial \theta_o} = -Q \frac{\partial \theta}{\partial t} \qquad (6')$$

or

$$\frac{\partial \theta}{\partial t} = \frac{\partial V}{\partial x} \sin \theta - \frac{\partial V}{\partial z} \cos \theta \ . \qquad (7)$$

Equation (7) governs θ as a function of t along the ray. The integral form of (7) is

$$\frac{\cos \theta}{\cos \theta_o} = \frac{V}{V_o} \exp\left(-\int_o^t \frac{\partial V}{\partial x} \sec \theta \, d\tau\right) \qquad (8)$$

Evidently, when $\partial V/\partial x$ is identically zero, that is, V is independent of x, Equation (8) is the Snell's Law equation for V a function of one variable (z) only.

Equation (5) is the analog to (6) and hence to Snell's Law for the wave front curves. Q is a measure of the change in arc length of the wave front per change in initial angle. It is related to the intensity

function along the ray relative to the intensity on a unit sphere. The intensity function is given approximately by

$$I \cong \frac{P(\Delta\theta_o)\cos\theta_o}{x(\Delta\sigma)}$$

where P is the intensity per unit area at a unit distance. Passing to the limit as $\Delta\theta_o \longrightarrow \theta$ gives

$$I = \frac{P\cos\theta_o}{x(t)Q(t)} . \tag{9}$$

Q(t) is an important measure of intensity fluctuation due to refraction. The differential equation for Q along a ray can be obtained from differentiating (5) with respect to t.

$$\frac{\partial^2 Q}{\partial t^2} - \left(\frac{2}{V}\frac{\partial V}{\partial t}\right)\frac{\partial Q}{\partial t} + V\left(\frac{\partial^2 V}{\partial z^2}\cos^2\theta + \frac{\partial^2 V}{\partial x^2}\sin^2\theta - 2\frac{\partial^2 V}{\partial x\partial z}\sin\theta\cos\theta\right)Q = 0 \tag{10}$$

The initial conditions are $Q = 0$ and $\partial Q/\partial t = V_o$ at $t = 0$. Equation (10) is equivalent to the differentiability condition

$$\frac{\partial^2\theta}{\partial\theta_o\partial t} = \frac{\partial^2\theta}{\partial t\partial\theta_o} .$$